

## OPTIMAL CONTROL OF PHYTOREMEDIATION TECHNIQUES FOR HEAVY METALS REMOVAL IN SHALLOW WATER

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**Abstract:** Term "heavy metal" usually denotes any dense metal that is marked for its potential toxicity, especially within environmental contexts. Heavy metals can be found naturally in the earth, but they usually become hazardous as a result of anthropogenic activities. Toxic metals tend to accumulate in living organisms as they are hard to metabolize and cannot be biodegraded. One of the main methods for remediation of heavy metal-contaminated waters is phytoremediation, that refers to the use of plants (algae in this particular case) to clean up water contaminated with hazardous chemicals. Phytoremediation takes advantage of the ability of algae to adsorb toxic heavy metals from the environment, resulting in higher concentrations than those in the surrounding water. In this work we deal with the optimization of different issues related to phytoremediation techniques, by combining mathematical modelling, optimal control of partial differential equations and numerical optimization (in the spirit of several previous works of the authors devoted to similar environmental control problems). In particular, we present a 2D mathematical model coupling the system for shallow water hydrodynamics with the system of nonlinear equations modelling the concentrations of heavy metals, algae and nutrients in large waterbodies. We also propose a full algorithm for computing the numerical solution of the system. In this paper we are interested in two main aspects: (i) determining the minimal quantity of algae to be used in the phytoremediation process, and (ii) locating the optimal place for such algae mass. These questions are formulated as optimal control problems for this scenario, and several numerical results for a realistic problem posed in "Ría de Vigo" (NW Spain) are presented.

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