

## **TOPOLOGY OPTIMIZATION WITH THE INERTIA RELIEF METHOD**

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**Abstract:** In many applications it is of interest to perform static finite element (FE) analyses on unconstrained bodies, including airplanes, helicopters and satellites in flight. This is particularly the case if topology optimization (TO) is to be used, since TO with time-dependent equations can be very computationally expensive. However, since the body is unconstrained it need not be in static equilibrium under the given loading, and therefore it is not a priori clear that a meaningful static analysis can be performed. But if one assumes that the loading is time-independent and the rotational part of the induced rigid body accelerations is small, then the inertial forces associated with such accelerations can be calculated exactly and subtracted from the given loading, making the net force and torque acting on the body equal zero. Neglecting vibrations, the body is now in static equilibrium; the method whereby this was achieved is sometimes called the inertia relief method. The body is still unconstrained however, so the equilibrium equation lacks a unique solution, since an arbitrary (linearized) rigid body motion can be added to one solution to get another. A common way to handle this problem of non-uniqueness is to impose constraints on the displacement (typically by prescribing displacement components at FE nodes) which prevent rigid motions while leaving the deformation unaltered. This method is simple but may have a significant negative impact on the conditioning of the stiffness matrix. Another, less frequently seen, way to avoid non-uniqueness is to add a penalty term to the equilibrium equation that forces any rigid motion to zero. The main drawback of this method is that the resulting system matrix is dense. Fortunately however, the structure of the matrix is such that matrix-vector products used by iterative linear solvers can be computed efficiently. In this work we formulate the inertia relief method for unconstrained, linearly elastic structures in an infinite-dimensional setting and use the model in topology optimization with the goal of maximizing stiffness. Discretization is carried out using the FE method, and convergence of the approximation scheme is shown. The optimization problem is posed in nested form and the FE equilibrium equation is solved using a multi-grid preconditioned conjugate gradient method. Numerical examples illustrate the idea and include comparisons of the above-mentioned methods for handling the singular stiffness matrix and the effect of different material interpolation schemes.