

## ADJOINT METHOD FOR TOPOLOGICAL DERIVATIVES FOR OPTIMIZATION TASKS WITH MATERIAL AND GEOMETRICAL NONLINEARITIES

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**Abstract:** The Topological Derivative (TD) is the sensitivity of a functional for the introduction of an infinitesimal hole into the mechanical body. For linear elasticity problems in structural mechanics the TD is known in a closed analytical form for various functionals. In this contribution, the TD for crash loaded structures is derived with the adjoint sensitivity analysis. Wherever the analytical derivation is limited, a numerical, so-called microscale, investigation cf. [1] and [2] completes the TD. The goal is to get a sensitivity for topology changes dealing with material and geometrical nonlinearities, inertial effects and time dependency. The main idea of the adjoint sensitivity analysis is to circumvent the direct calculation of the sensitivity of the displacement field. Instead, the adjoint equilibrium equation has to be solved. In this approach, material derivation [3] and partial integration in the time domain cf. [4] are applied to the TD. This ensures, that the inertial effects are kept, as they are important for a reliable crash simulation. The result is a backward integration scheme for the adjoint state. For the calculation of the TD, only the boundary integral on the introduced hole remains. This integral contains terms of the functional and the weak equilibrium condition. Corresponding to [1], a microscale investigation for each term replaces the analytical form with meta-models. The presented scheme to derive the TD will be illustrated with an academic example. For a functional that copes with crash relevant requirements, the TD is shown and compared with the sensitivities derived with numerical differentiation. The numerical effort and the storage requirement will be outlined, as well as the chances and limitations dealing with crash software for the calculation of the adjoint state.

### References

- [1] K. Weider and A. Schumacher, *On the calculation of Topological Derivatives considering an exemplary nonlinear material model*, PAMM, Vol. 16<sup>(1)</sup>, 717-718 (2016).
- [2] K. Weider and A. Schumacher, *A topology optimization scheme for crash loaded structures using Topological Derivatives*, *Advances in Structural and Multidisciplinary Optimization*, WCSMO 2017, 1601-1614 (2018).
- [3] A. A. Novotny, J. Sokolowski, *Topological Derivatives in Shape Optimization*, Springer-Verlag Berlin Heidelberg (2013).
- [4] J. Dahl, J. S. Jensen, O. Sigmund, *Topology optimization for transient wave propagation problems in one dimension*, *Struct Multidisc Optim*, Vol. 36, 585-595 (2008).