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THEORY OF GRILLAGE OPTIMIZATION – A DISCRETE SETTING

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Abstract: Classically, a grillage structure can be viewed as a finite, planar system of beams under bending that meet at joints. An alternative, continuous description revolves around a fibrous plate with the fibers representing infinitely thin beams. The topic of optimum grillages in the latter setting gained interest in the early '70s; numerous analytical solutions have been derived ever since. Meanwhile, a far more popular theory of optimum frameworks called Michell trusses has been extensively studied since the very beginning of the 20th century. An efficient method for discretizing the Michell problem was developed in the '60s and it employed the so-called ground structure - a dense, yet finite network of truss bars that interconnects a fixed grid of nodes. Then the design variables are chosen as the cross-section areas constant per each bar which renders the optimization problem as discrete at its root. Later, this problem, set either as plastic or elastic design, received a thorough mathematical treatment and was reduced to a pair of mutually dual linear programming problems: kinematic and equilibrium forms with, respectively, nodal displacements and with member forces as variables. Hence, due to linearity, the truss optimization problem naturally admits a neat matrix-vector formulation. Since the continuous grillage optimization problem mathematically lies in close proximity to Michell problem, possibility of adopting the ground structure approach for numerical optimization of grillages follows. This was done first in the early '90s combined with FEM for elastic design problem, recently also for large scale plastic design problems. Although the methods developed proved to produce satisfactory numerical grillage layouts, the discrete problem being solved was in fact an approximated version of the true grillage-ground-structure optimization problem. As opposed to truss problem, the optimum width distribution in a grillage may vary along the beams and so may the bending moment function. Hence, reducing the problem to an exact, discrete (finite dimensional) setting is not straightforward and is set as a goal of the present work. In this paper the point of departure is a pair of mutually dual variational problems written for the grillage ground structure: kinematic and equilibrium forms with, respectively, displacement and bending moment functions as variables. The two infinite dimensional problems are then analytically reduced to the two mutually dual discrete forms; the matrix-vector fashion known from truss optimization is preserved. One of the key results is an appearance of the two dual norms defined on the plane.